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An Extended PID Control Framework

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Abstract— **Proportional-integral-derivative (PID) control is the most prevalent form of feedback control for a wide range of real physical applications. Traditional PID control theory is built on transfer function models. In this paper, we try to extend PID control to state-space models. Two formulations are presented to form an extended PID (EPID) control framework. One is the proportional-integral tracking controller (PITC), which includes a proportional and an integral part of all state errors. The other is the adaptive feedforward tracking controller (AFTC), which consists of a feedback part of state errors and a feedforward part which is obtained adaptively by using the previous sampled input. An interesting observation is that the two extended PID formulations are shown to be equivalent. EPID provides us a new perspective to view the mechanism of PID control itself as well as its relationship with other control theories such as tracking control, iterative learning control, and disturbance observer. All the points are demonstrated through a cart-pendulum example.**

I. Introduction

PROPORTIONAL-INTEGRAL-DERIVATIVE (PID) control is the most widely used controller today. A PID controller is a sum of three components, that is, the proportional, integral, and derivative of the output error, which represents the present, the past, and the future information of the system, respectively. Due to its simplicity, PID can be easily understood by control engineers and applied to a wide variety of real-world control problems. It is estimated that over 90% of industrial control loops are using PID control [1].

The history of PID control can date back to Elmer Sperry's ship autopilot in 1910. Over the last century, numerous works have been done on PID control in both academic and industrial fields. A detailed summarization can be found in the review articles [2-4] and related books [5-6]. As can be observed in [2-6], studies on PID control are mostly about tuning rules, identification schemes, and adaptation techniques. With the development of linear control [7], optimal control [8], nonlinear control [9], adaptive control [10], robust control [11], etc., modern control theory has had a rapid development. Studying PID control in a modern perspective then becomes an interesting topic. Reference [12] gives an analysis on PID controller design for second order nonlinear uncertain systems. And the recently developed control technique, active disturbance rejection control (ADRC) [13-14] also inherits from PID.

To date, the research on PID control are primarily based on transfer functions. As we know, a common formulation for PID control is usually written as $u = k_p e + k_i \int_0^t e(\tau)\, d\tau + k_d \dot{e}$, where u is the control input, e is the output tracking error, and k_p, k_i, k_d are gains for the proportional, integral, and derivative part, respectively. However, this classical PID formulation does have some drawbacks. Firstly, it is directed at single input single output (SISO) systems. Although some efforts have been made to extend it to multi-input multi-output (MIMO) systems [16-17]. None of them have given a generic formulation that is widely accepted as far as we know. Secondly, PID control treats all systems the same without considering the system characteristics, such as the system order. Such ignored system features may influence the control performance.

In modern control theory, a landmark is the state-space approach [15] pioneered by Kalman, which describes the system dynamics by state variables. An advantage of this description is that it reflects the complete information of all the system states. In this paper, an extended PID (EPID) control framework is proposed based on the state-space models. Specifically, two equivalent formulations of EPID are investigated, including proportional-integral tracking controller (PITC) and adaptive feedforward tracking controller (AFTC). Based on this framework, some new insights are given for PID control. First, it is found that for minimum phase systems, the integral gain might be set very high without breaking the system stability. And a high integral gain helps improve the tracking accuracy and reject disturbances. Second, EPID also reveals the relationship between PID control and other control theories, including tracking control, iterative learning control, and disturbance observer. The contribution of this paper is the extension of PID control to state space. EPID gives a unified PID control framework for SISO/MIMO and lower-order/higher-order systems, where PI and PID control are special cases of EPID for first-order and second-order SISO systems, respectively.

The rest of this paper are organized as follows. Section II introduces PITC. Section III investigates the features of a high integral gain. Section IV introduces AFTC. Section V summarizes the proposed EPID framework. Section VI gives the simulation results of a cart-pendulum system and Section VII concludes this paper.

II. From PID to PITC

The classical PID control theory is based on the transfer function description of system models. In this section, an extended PID formulation is developed by using the state-space description of system models.

In linear control theory, a system is expressed in a state-variable form

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$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{n\times n}, \mathbf{B} \in \mathbb{R}^{n\times m}, \mathbf{C} \in \mathbb{R}^{m\times n}$ are system matrices, $\mathbf{x} \in \mathbb{R}^{n}$ is the state vector, $\mathbf{u}, \mathbf{y} \in \mathbb{R}^{m}$ are the input vector and output vector, respectively. Compared to the transfer function description, this model description is more complete since it reflects all the state information. For this system, a typical feedback controller is designed as follows

$$\mathbf{u} = \mathbf{K}\mathbf{x} \tag{2}$$

where $\mathbf{K} \in \mathbb{R}^{m\times n}$ is the control gain matrix. It leads to the following closed-loop system

$$\dot{\mathbf{x}} = \left(\mathbf{A} + \mathbf{B}\mathbf{K}\right)\mathbf{x} \tag{3}$$

It is well known that the pole assignment method can be used to design the feedback gain to make the closed-loop system asymptotically stable if system (1) is controllable.

More generally, this controller can also be applied to tracking control with minor modification. Suppose the desired output reference is $\mathbf{y}_\mathbf{r}$. Define the state reference $\mathbf{x}_\mathbf{r}$ and the ideal input $\mathbf{u}_\mathbf{r}$ as the solution of the following dynamic equation

$$\dot{\mathbf{x}}_\mathbf{r} = \mathbf{A}\mathbf{x}_\mathbf{r} + \mathbf{B}\mathbf{u}_\mathbf{r}$$
$$\mathbf{y}_\mathbf{r} = \mathbf{C}\mathbf{x}_\mathbf{r} \tag{4}$$

Denote $\mathbf{e} = \mathbf{x} - \mathbf{x}_\mathbf{r}$ as the state error. Subtract (4) from (1) gives the error dynamics

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\left(\mathbf{u} - \mathbf{u}_\mathbf{r}\right) \tag{5}$$

Then the tracking controller can be designed as

$$\mathbf{u} = \mathbf{u}_\mathbf{r} + \mathbf{K}\left(\mathbf{x} - \mathbf{x}_\mathbf{r}\right) \tag{6}$$

Substituting (6) into (5) follows that

$$\dot{\mathbf{e}} = \left(\mathbf{A} + \mathbf{B}\mathbf{K}\right)\mathbf{e} \tag{7}$$

which has the same form as (3).

For convenience, we will call (6) as the linear tracking controller, which is widely used in inversion-based control [18-19] and output regulation [20-21]. The linear tracking controller not only applies to linear systems, but also can achieve asymptotic tracking for nonlinear systems. Its exact output tracking capability is benefited from the exact solution of the feedforward $\mathbf{u}_\mathbf{r}$ and the state reference $\mathbf{x}_\mathbf{r}$, which are the key for exact tracking. This controller can be interpreted as: the feedforward $\mathbf{u}_\mathbf{r}$ provides the needed control input for exact tracking, while the feedback $\mathbf{K}\left(\mathbf{x} - \mathbf{x}_\mathbf{r}\right)$ provides correction when the system states are deviated from the state reference and thus keeps the system locally stable along the desired trajectory.

It is noted that all state errors are used for feedback in the linear tracking controller (6). However, it is not always easy to obtain the feedforward $\mathbf{u}_\mathbf{r}$ and the state reference $\mathbf{x}_\mathbf{r}$ from the output reference $\mathbf{y}_\mathbf{r}$, especially for nonminimum phase systems where the internal state reference should be a bounded solution of the unstable zero dynamics [18-19]. For convenience, we will focus on minimum phase systems in this paper.

Consider a MIMO minimum phase system with m inputs and m outputs. Denote $\mathbf{u} = \left[u_1, u_2, ..., u_m\right]^T$ as the input vector, $\mathbf{y} = \left[y_1, y_2, ..., y_m\right]^T$ as the output vector. Suppose the relative degree for the output is $\left\{r_1, r_2, ..., r_m\right\}$, which represents the order of differentiations for the output when any input appears. Then the input-output dynamics can be written as follows:

$$\mathbf{y}^{(r)} = \mathbf{F}\left(\mathbf{x}, \mathbf{u}\right) + \mathbf{d} \tag{8}$$

where $\mathbf{y}^{(r)}$ is defined as $\mathbf{y}^{(r)} = \left[y_1^{(r_1)}, y_2^{(r_2)}, \ . \ . \ . \ , y_m^{(r_m)}\right]^T$, $\mathbf{x} = \left[y_1, \dot{y}_1, ..., y_1^{(r_1-1)}, ..., y_m, \dot{y}_m, ..., y_m^{(r_m-1)}\right]^T$ is the state vector, and $\mathbf{d} = \left[d_1, d_2, ..., d_m\right]^T$ represents some matched disturbances. The system may also have internal dynamics which are not reflected in the input-output dynamics. However, they do not need to be taken care of since they are stable for minimum phase systems and thus can be treated the same as the disturbance $\mathbf{d}$.

For this system, it has a very good property that once the output reference $\mathbf{y}_\mathbf{r}(t) = \left[y_{1r}(t), y_{2r}(t), ..., y_{mr}(t)\right]^T$ are given, then all the state references are known, that is, $\mathbf{x}_\mathbf{r} = \left[y_{1r}, \dot{y}_{1r}, ..., y_{1r}^{(r_1-1)}, ..., y_{mr}, \dot{y}_{mr}, ..., y_{mr}^{(r_m-1)}\right]^T$, which is independent of the system model. Therefore, all the state tracking error $\mathbf{e} = \mathbf{x} - \mathbf{x}_\mathbf{r}$ are available for feedback control.

For system (8), by using state error instead of output error, the PID controller can be written in the following form

$$\mathbf{u} = \mathbf{K}_\mathbf{p}\left(\mathbf{x} - \mathbf{x}_\mathbf{r}\right) + \frac{1}{T_i}\int_0^t \mathbf{K}_\mathbf{p}\left(\mathbf{x} - \mathbf{x}_\mathbf{r}\right) d\tau \tag{9}$$

where $\mathbf{K}_\mathbf{p} \in \mathbb{R}^{m\times n}$ is the proportional gain matrix and T_i is the integral time (integral gain defined as $k_i = 1/T_i$). It should be noted that the derivative part is not missing, it is actually contained in the proportional part since the state error vector $\mathbf{x} - \mathbf{x}_\mathbf{r}$ contains the derivative of the output error when the system relative degree is bigger than 1.

The controller (9) has the same structure as a PI controller, so we call it proportional-integral tracking controller (PITC). Compared to a PID controller, the proportional gain in PITC becomes a matrix while the integral time is still a scalar parameter. PITC takes advantage of the state-variable description, which allows it to be applied to both SISO and MIMO, lower-order and higher-order systems. It is also compatible with the classical PID formulation since that PI and PID control can be seen as particular cases of PITC for first-order and second-order SISO systems, respectively.

III. The Features of A High Integral Gain

High-gain feedback has been studied in a variety of articles [22-24]. However, less attention has been paid on high-integral-gain control [25]. As we know, the integral term in PID control is very special. It can compensate the system uncertainties automatically and leads to zero steady-state error for constant reference/disturbance. This section will study the features of a high integral gain through two typical examples.

Consider a typical first-order system $\dot{x} = u + d$, where d is the external disturbance. Denote the tracking error as $e = x - x_r$ where x_r is the reference trajectory. Using the PITC

$$u = -k_p e - k_i \int_0^t k_p e d\tau \tag{10}$$

where $k_p, k_i > 0$. The closed-loop system becomes

$$\dot{e} = -k_p e - k_i \int_0^t k_p e d\tau + d - \dot{x}_r \tag{11}$$

Consider $d - \dot{x}_r$ as a lumped disturbance, then the transfer function of the closed-loop system from $d - \dot{x}_r$ to e is obtained as

$$\frac{E(s)}{D(s)} = \frac{s}{s^2 + k_p s + k_p k_i} \tag{12}$$

Since the parameters satisfy $k_p, k_i > 0$, it is easy to verify that the closed-loop system is stable and k_i can be as large as we want.

Similarly, for a typical second-order system $\ddot{x} = u + d$, using the PITC

$$\begin{aligned} u &= -k_p e - k_d \dot{e} + k_i \int_0^t \left(-k_p e - k_d \dot{e}\right) d\tau \\ &= -\left(k_p + k_d k_i\right) e - k_d \dot{e} - k_p k_i \int_0^t e d\tau \end{aligned} \tag{13}$$

where $k_p, k_i, k_d > 0$. The closed-loop system becomes

$$\ddot{e} = -k_d \dot{e} - \left(k_p + k_d k_i\right) e - k_p k_i \int_0^t e d\tau + d - \ddot{x}_r \tag{14}$$

The transfer function of the closed-loop system from the lumped disturbance $d - \ddot{x}_r$ to e is obtained as

$$\frac{E(s)}{D(s)} = \frac{s}{s^3 + k_d s^2 + \left(k_p + k_d k_i\right) s + k_p k_i} \tag{15}$$

It is easy to verify that the closed-loop system is stable if the parameters satisfy $k_p, k_i, k_d > 0$ and $k_p k_d > k_i \left(k_p - k_d^2\right)$. Moreover, if we let $k_p < k_d^2$, then k_i can be infinitely large without breaking the stability of the closed-loop system.

It is assumed that the lumped disturbance is bounded so that the tracking error is also bounded. Then Bode diagram can be used to evaluate the influence of the lumped disturbance on the tracking error. We are interested in the influence of the integral gain. Therefore, we fix k_p, k_d and change k_i to see the differences. By selecting $k_p = 2$ for the first-order system and $k_p = k_d = 4$ for the second-order system, the magnitude responses of the transfer functions (12) and (15) are shown in Fig. 1 (a) and (b), respectively.

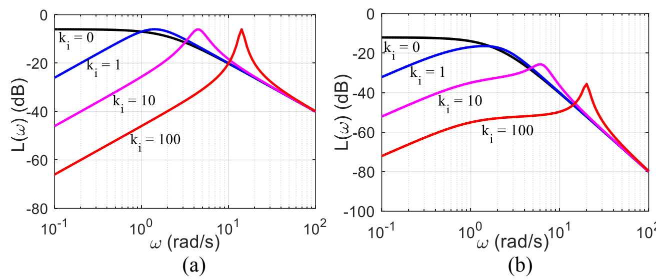

Figure 1. Bode diagram from lumped disturbance to tracking error for (a) first-order system (b) second-order system.

It can be seen that both figures show the same trend, that is, as the integral gain increases, the frequency corresponds to the peak point moves to the right and the magnitude response has a significant drop at low frequency. Since the lumped disturbance is composed of the output reference and the external disturbance, it indicates that a high integral gain is beneficial to suppress low-frequency disturbances and track slow-varying signals. With a relatively high integral gain, the magnitude response can be made very low at a wide frequency range, so that the influence of the disturbance can be neglected, and thus precision tracking can be achieved. In a sense, the integral part with a high integral gain is very similar to a disturbance observer [26], whose function is to compensate the disturbance.

To sum up, this section has shown two important features of a high integral gain. First, it shows that for minimum phase systems, the integral gain might be set very high without breaking the system stability. Second, a high integral gain helps improve tracking accuracy and reject disturbance. Although the conclusions are obtained from two simple examples, they may also work for more complicated nonlinear systems, which is verified later through a cart-pendulum system.

IV. From PITC to AFTC

In this section, a transformation is made on PITC to get another extended PID formulation. For the PITC formulation (9), the following approximation holds when T_i is small

$$\begin{aligned} \mathbf{u} &= \mathbf{K_p e} + \frac{1}{T_i}\int_0^t \mathbf{K_p e} d\tau \\ &= \mathbf{K_p e} + \frac{1}{T_i}\left[\int_0^{t-T_i} \mathbf{K_p e} d\tau + \int_{t-T_i}^t \mathbf{K_p e} d\tau\right] \\ &\approx \mathbf{K_p e} + \frac{1}{T_i}\left[\int_0^{t-T_i} \mathbf{K_p e} d\tau + T_i \mathbf{K_p e}\left(t - T_i\right)\right] \\ &= \mathbf{K_p e} + \left[\mathbf{K_p e}\left(t - T_i\right) + \frac{1}{T_i}\int_0^{t-T_i} \mathbf{K_p e} d\tau\right] \\ &= \mathbf{K_p e} + \mathbf{u}\left(t - T_i\right) \end{aligned} \tag{16}$$

Therefore, it leads to another EPID formulation

$$\mathbf{u} = \mathbf{u}(t - T_i) + \mathbf{K}_\mathbf{p}\mathbf{e} \tag{17}$$

In this formulation, the input is a sum of the previous input with a small time delay plus the current error feedback.

The formulation (17) is very simple but it reflects some deep mechanism. First, it is very similar to the linear tracking controller $\mathbf{u} = \mathbf{u}_\mathbf{r} + \mathbf{Ke}$, only with the feedforward $\mathbf{u}_\mathbf{r}$ replaced by the delayed input $\mathbf{u}(t - T_i)$. Therefore, it can be speculated that the delayed input actually plays the role as a feedforward. The difference is that the feedforward in (17) is obtained adaptively by using the previous input, which does not require exact model knowledge. Therefore, we will call formulation (17) adaptive feedforward tracking controller (AFTC).

What's more, another controller, the iterative learning control [27-29] also has a very similar form. Iterative learning control is used for a repeated process, which generates a feedforward control that tracks a specific reference or rejects a repeating disturbance by learning from several iterations. The simplest iterative learning controller [29] is

$$\mathbf{u}(t, k) = \mathbf{u}(t, k-1) + \mathbf{Ke}(t, k-1) \tag{18}$$

where k is the repetition index. It can be seen that the input in the current operation is determined by the input of the previous operation plus the proportional contribution of the tracking error in the previous operation. As the operation repeats, the input will converge to the ideal input that yields an exact tracking. If we discretize the AFTC (17), it becomes

$$\mathbf{u}(t_k) = \mathbf{u}(t_{k-1}) + \mathbf{K}_\mathbf{p}\mathbf{e}(t_k) \tag{19}$$

where $t_k = kT_i$ is the discrete time step. It can be observed that the current input is determined by the input of the previous time step plus the proportional contribution of the tracking error in the current time step, which is very similar to (18). Compared to the iterative learning controller (18), AFTC is much more efficient since it learns in real time, with no need to repeat the process for several times.

V. THE EPID FRAMEWORK

In this section, the EPID framework and its relationship with other control techniques are summarized. As shown in Fig. 2, EPID includes PITC and AFTC, where PID is a particular case of PITC.

The integral part in PITC is equivalent to the feedforward part in AFTC, whose function is holding; while the proportional part in PITC corresponds to the feedback part in AFTC, whose function is correcting. This also gives us another perspective to view the three components in PID control. The integral part which uses the past information actually plays the role as a feedforward, which is a kind of prediction; while the derivative part, which used to be thought as a prediction, is an ordinary state feedback which has the same role as the proportional part in PITC.

EPID is also closely related to many other control techniques, including tracking control, iterative learning control, and disturbance observer technique.

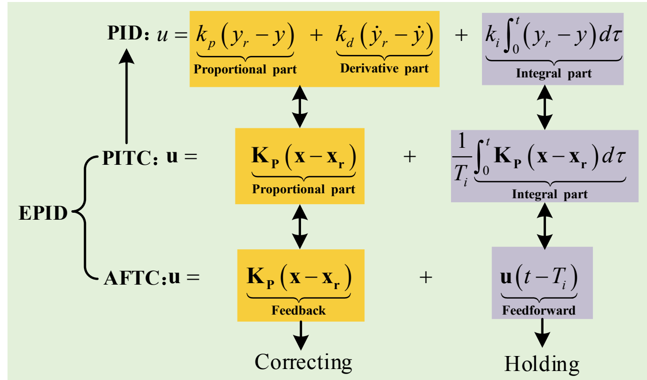

Figure 2. The EPID framework.

EPID and PID: EPID is an extended version of PID control. EPID takes advantage of the state-space model description and replaces the output error in PID control by state error. PI and PID control are particular cases of PITC for first-order and second-order SISO systems, respectively.

EPID and Iterative Learning Control: AFTC and iterative learning control [27-29] have similar control structure. The feedforward in AFTC is also obtained iteratively. Compared to iterative learning control, EPID is learning in real time, with no need to repeat the process for several times.

EPID and Tracking Control: AFTC also has similar structure to the linear tracking controller (6) which is widely used in inversion-based control [18-19] and output regulation [20-21]. However, both inversion-based control and output regulation depend on system model to find out the feedforward signal. In output regulation theory, the feedforward is obtained by solving the regulation equation; while in inversion-based control theory, the feedforward is obtained from model inversion. Thus both of them are sensitive to model uncertainties and disturbances. But in EPID, the feedforward is obtained adaptively without need to know exact model knowledge.

EPID and Disturbance Observer: EPID is a special type of disturbance observer-based controller [26]. The integral part plays the role as a disturbance observer, but it must cooperate with the proportional part and cannot work independently.

In a word, EPID not only extends the PID control framework, but also builds a bridge between different control techniques which is important for us to understand their inner relationships.

VI. THE CART-PENDULUM EXAMPLE

A cart-pendulum system is shown in Fig. 3, which is composed of a cart that can move in the horizontal plane and a passive pendulum that can rotate freely along the pivot.

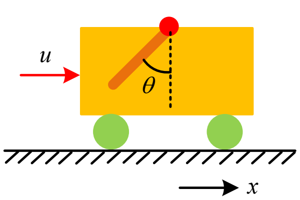

Figure 3. The cart-pendulum system.

In fig.3, u is an external force imposed on the cart, x is the cart position, θ is the pendulum angle, m and M are the mass of the pendulum and the cart, respectively, l is the half length of the pendulum. The equations of motion are

$$\begin{aligned}
\dot{x} &= v \\
\dot{v} &= \frac{3mg\sin\theta\cos\theta + 4ml\omega^2\sin\theta + 4u}{4(M+m) - 3m\cos^2\theta} \\
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{-3(M+m)g\sin\theta - 3ml\omega^2\sin\theta\cos\theta - 3\cos\theta u}{4(M+m)l - 3ml\cos^2\theta}
\end{aligned} \tag{20}$$

For this system, we select the output as the cart position. *Particularly, we only consider the control of the cart motion while the pendulum motion is treated as a disturbance* so that θ, ω will not be used in the controller. Therefore, the system state vector is $\mathbf{x} = [x, v]^T$.

In the simulation, the model parameters are selected as $m = 1kg$, $M = 10kg$, $l = 1m$, and $g = 9.8m/s^2$. The proportional gain is designed as $\mathbf{K_p} = [-2, -20]$ and the integral time will use different values. The initial condition is set to zero for all states. As a practical consideration, the control input is assumed to be limited in the range $u \in [-30, 30]$. For a sinusoidal output reference $x_r = \sin t$, the simulation results are shown in Fig. 4 and Fig. 5.

From Fig. 4, it can be observed that the system keeps stable as k_i changes from 0 to 50. The tracking error is big when the integral action is turned off ($k_i = 0$). As k_i increases, the tracking error becomes smaller. When $k_i = 50$, the tracking error is close to zero, which, however, is at the sacrifice of a serious input chattering at the beginning as can be observed from Fig. 5, which is due to the initial condition mismatch ($v(0) = 0$ while $v_r(0) = 1$). In practice, a proper integral time should be selected to ensure a good performance (integral gain not too small) and an acceptable input chattering (integral gain not too big).

Furthermore, we fix $k_i = 50$ and design a combined reference. The simulation results are shown in Figs. 6-8. Fig. 6 shows the cart position and the pendulum angle. It can be seen that the cart position tracks very well with the reference, regardless of the disturbance from the pendulum swing. Fig. 7 shows the tracking error and the control input. It can be seen that a serious chattering happens for both the control input and the tracking error at t = 5, 15, 25, and 40 s. For t = 5 and 15 s, the ideal input u_r is continuous, but the state reference v_r has a step change, which makes the input deviates from the ideal input and takes some time to recover. For t = 25 and 40 s, $\ddot{x}_r$ has a step change which results in a step change in the ideal input, and the input takes a transient adjust to catch up. A dynamic equilibrium is reached after each transient adjustment, where the input keeps close to the ideal input and the tracking error varies smoothly. Fig. 8 shows the feedback and feedforward components of the control input. It can be observed that the feedback works actively to correct the input in the discontinuous points, which causes a chattering in the feedforward. Then the feedback keeps very small at dynamic equilibrium and the input is dominated by the feedforward.

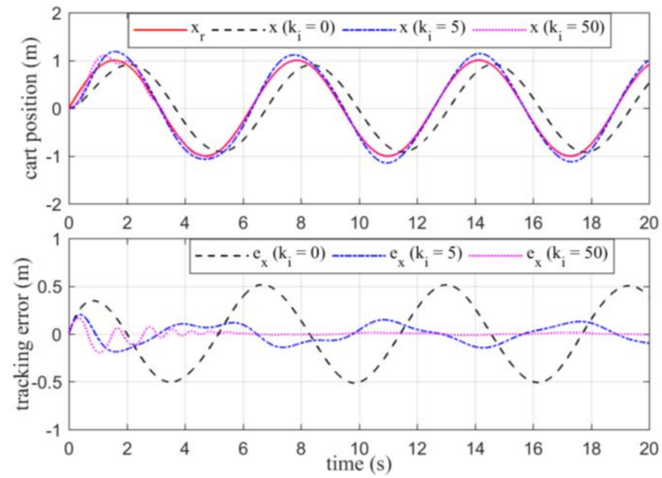

Figure 4. Cart position and tracking error for a sinusoidal reference.

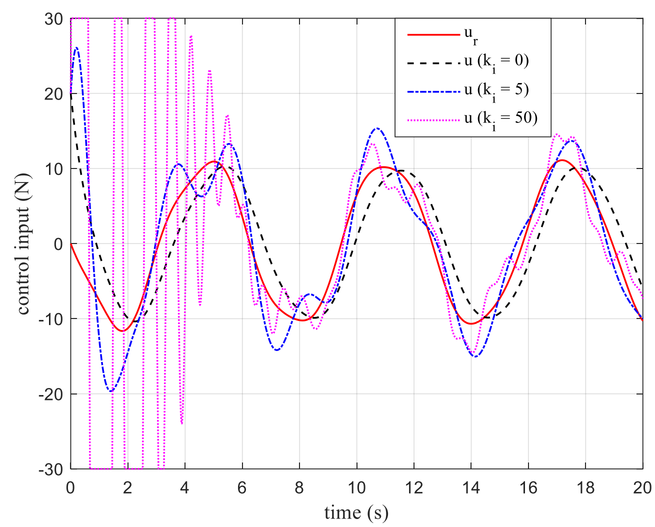

Figure 5. Control input for a sinusoidal reference.

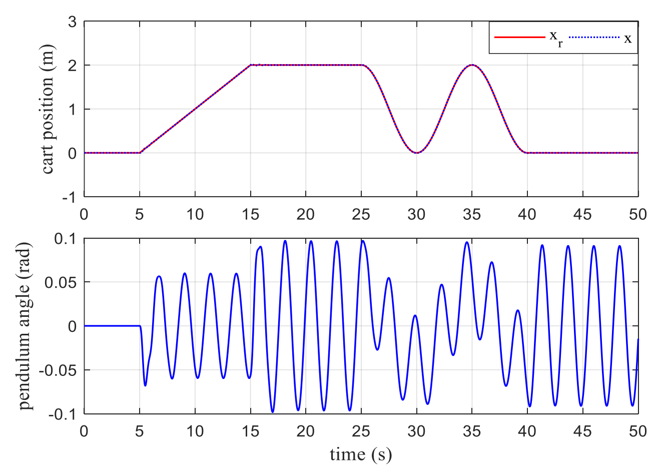

Figure 6. Cart position and pendulum angle for a combined reference.

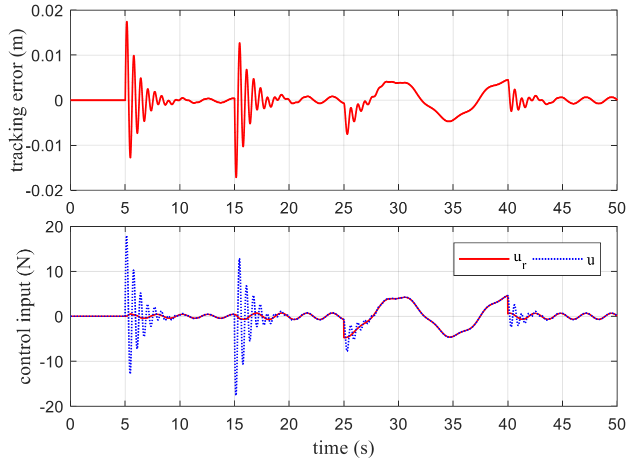

Figure 7. Tracking error and control input for a combined reference. The ideal input u_r is obtained from model inversion.

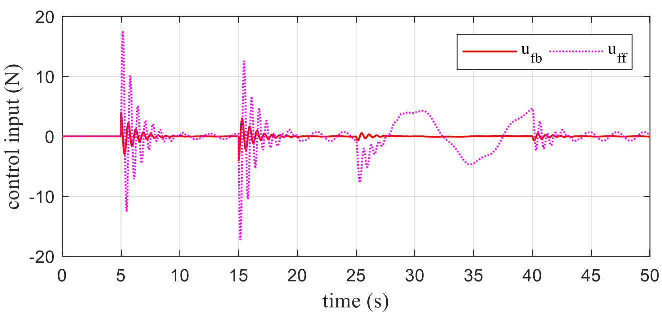

Figure 8. The feedback and feedforward components of the control input. The two parts are defined as $\mathbf{u}_{\mathbf{fb}} = \mathbf{K}_{\mathbf{p}}\mathbf{e}$ and $\mathbf{u}_{\mathbf{ff}} = \mathbf{u}(t-T_i) = k_i\int_0^t \mathbf{K}_{\mathbf{p}}\mathbf{e}d\tau$, respectively.

VII. Conclusions

This paper develops an extended PID (EPID) control framework with two formulations. EPID takes advantage of the state-space description. On one hand, EPID retains the simplicity of PID control. On the other hand, EPID makes full use of the state information and is more specified for different systems, such as SISO/MIMO and lower-order/higher-order systems. We highlight the function of a high integral gain, which is important to improve the tracking accuracy and reject disturbances. EPID gives us some new insights on PID control and reveals the relationship between PID control and many other control techniques, which displays a very broad view. However, like PID control, the stability of EPID may not be proved in a rigorous way, which will be left for exploration in the future.

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